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The field of reals with a predicate for the real algebraic numbers and a predicate for the integer powers of two. (English. English summary)

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The paper under review is a nice contribution to the study and classification of expansions of the real field. Let $\mathcal{R} = (\mathbb{R}, <, +, \cdot, \dots)$ be a polynomially bounded o-minimal expansion of the real field with field of exponents $\mathbb{Q}$, and let $\mathcal{S}$ be a proper elementary substructure whose underlying set S is dense in the usual order topology on $\mathbb{R}$. Denote the set of all integer powers of 2 by $2^{\mathbb{Z}}$. "Definable" will mean definable with parameters.

The main result can be stated as follows: Every subset of $\mathbb{R}^l$ definable in $(\mathcal{R}, \mathcal{S}, 2^{\mathbb{Z}})$ is a boolean combination of sets of the form

$$\{x \in \mathbb{R}^l : \exists y \in S^m \exists z \in (2^{\mathbb{Z}})^n \text{ such that } (x, y, z) \in W\}$$

for some $W \subseteq \mathbb{R}^{l+m+n}$ definable in $\mathcal{R}$. Among other things, it follows that every open definable set in $(\mathcal{R}, \mathcal{S}, 2^{\mathbb{Z}})$ is definable in $(\mathcal{R}, 2^{\mathbb{Z}})$. In addition, the author presents a natural axiomatization of the theory of $(\mathcal{R}, \mathcal{S}, 2^{\mathbb{Z}})$, which does not depend on the particular choice of $\mathcal{S}$, and proves that it has NIP.

Note that the assumption that the field of exponents of $\mathcal{R}$ is $\mathbb{Q}$ is necessary [see P. Hieronymi, Proc. Amer. Math. Soc. **138** (2010), no. 6, 2163–2168 (Corollary 1.5); MR2596055]. Similar results are known for $(\mathcal{R}, \mathcal{S})$ [L. van den Dries, Fund. Math. **157** (1998), no. 1, 61–78; MR1623615] and for $(\mathcal{R}, 2^{\mathbb{Z}})$ [C. L. Miller, in *Logic Colloquium '01*, 281–316, Lect. Notes Log., 20, Assoc. Symbol. Logic, Urbana, IL, 2005; MR2143901]. Indeed, the proof in the paper under review can be described as an amalgamation of the proofs of those two results. Comparable results for the real field expanded by a predicate for $2^{\mathbb{Z}}3^{\mathbb{Z}}$ and a predicate for $2^{\mathbb{Z}}$ were established in [A. Günaydın, *Model theory of fields with multiplicative groups*, Ph.D. thesis, Univ. Illinois Urbana-Champaign, 2008; MR2712584].

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.